October Math Gems

Problem of the week 24

§1 Problems

Problem 1.1. Solve the Diophantine equation $x^5 - y^5 = 1993$

Problem 1.2. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$ then $\tan^2 x =$?

Problem 1.3. $\sin^{100}(x) + \cos^{100}(x) = 1$ Solve for the value of x

Problem 1.4. $a + b = 6\sqrt{ab}$. Find $\frac{a}{b}$.

Problem 1.5. $\log_{4x} x + \log_{\frac{x}{2}} x = 2$. Solve for x.

Problem 1.6. Solve the exponential equation $2^x - 3^x = \sqrt{6^x - 9^x}$

Problem 1.7. The following product can be expanded into a power series with coefficients a_k:

$\prod_{n=1}^{100} \left[\frac{x^3}{4^n} + \left(\frac{\pi}{2^n}\right)^2\right] = \sum_{k=0}^{\infty} a_k x^k$

Find the coefficients a_k in front of the individual x^k terms for all $k \in N$

Problem 1.8. Determine all x that solve the equation $x^{2x} + 27^2 = 54x^x$

Problem 1.9. Find the value of this infinite sum: $\sum_{n=0}^{\infty} \frac{n^{2n}+2^n}{2^{3n}}$

Problem 1.10. If $a = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + ...$ Find the value of a

Problem 1.11. Find $f(x)$ for $f\left(\frac{x-3}{x+1}\right) + f\left(\frac{x+3}{1-x}\right) = x$

Problem 1.12. Solve the equation $x^4 - 8x + 63 = 0$

Problem 1.13. Let x and y be real numbers satisfying

$\left(x^2 + x - 1\right)\left(x^2 - x + 1\right) = 2\left(y^3 - 2\sqrt{5} - 1\right)$ and
$\left(y^2 + y - 1\right)\left(y^2 - y + 1\right) = 2\left(x^3 + 2\sqrt{5} - 1\right)$

Find $8x^2 + 4y^3$

Problem 1.14. Find positive integer n so that $\frac{80-6\sqrt{n}}{n}$ is the reciprocal of $\frac{80+6\sqrt{n}}{n}$

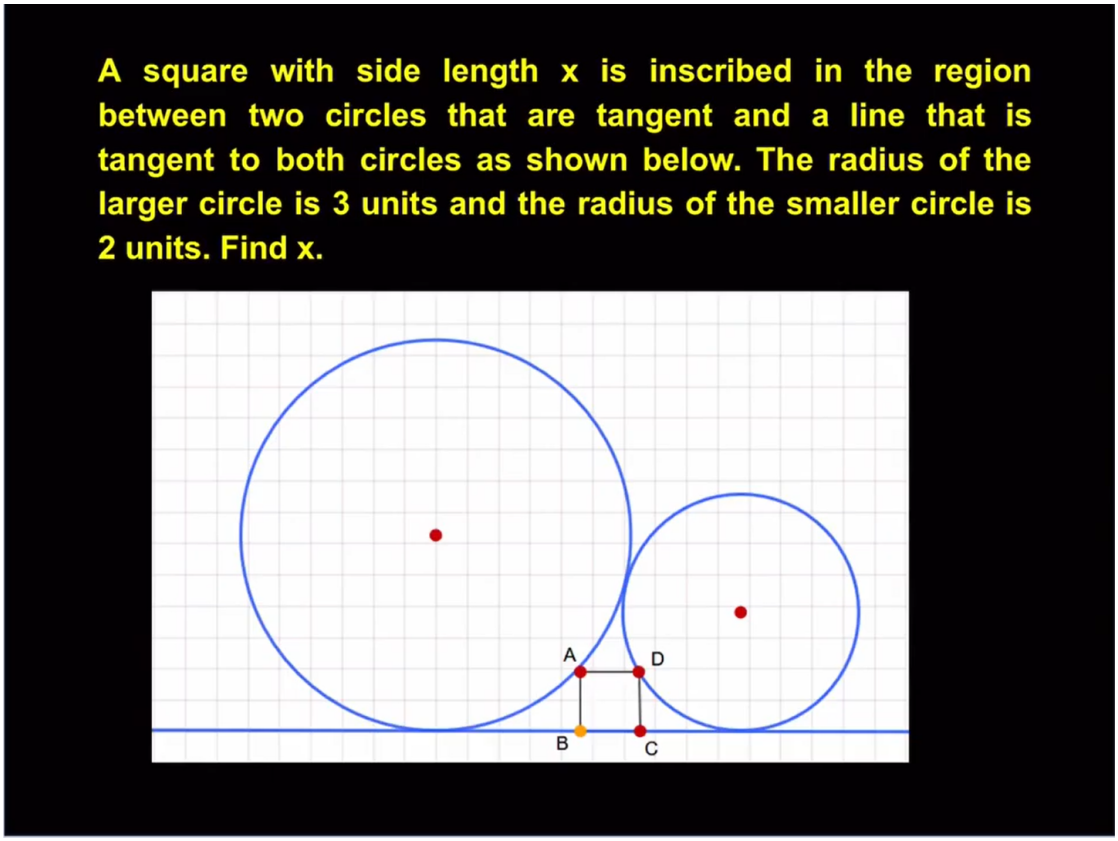
Problem 1.15. If a, b, c are non negative integers and $2^a + 2^b = c!$

Solve for a , b and c values

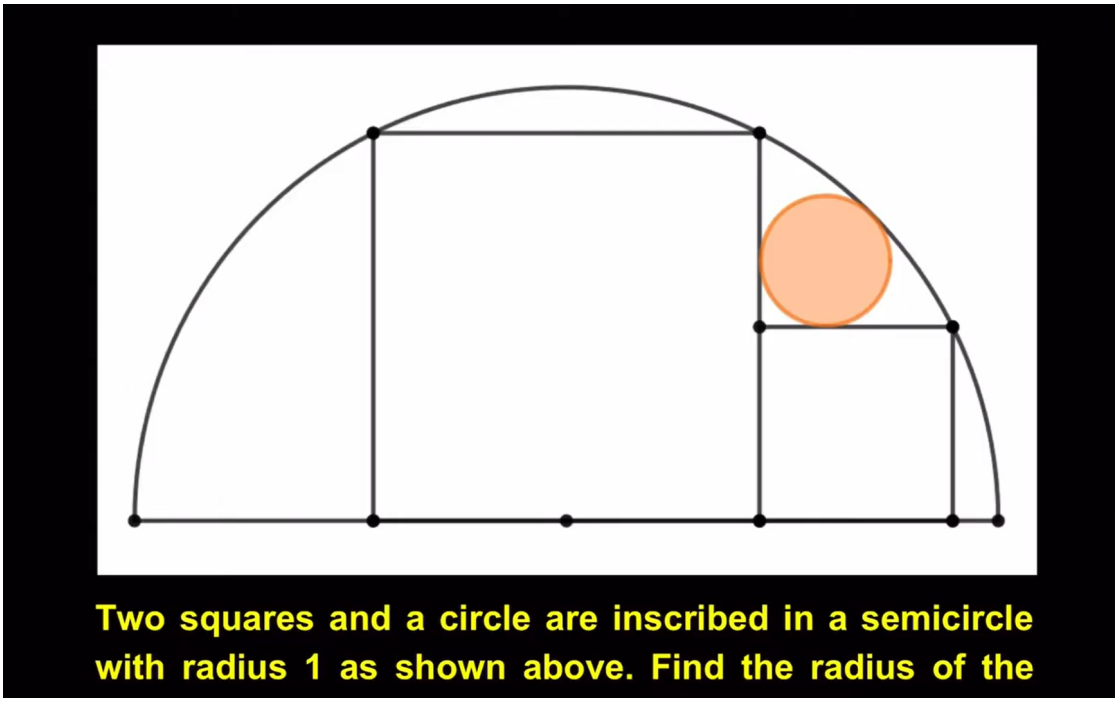
Problem 1.16. Solve for integer solutions of n

$n! = n^3 - n$

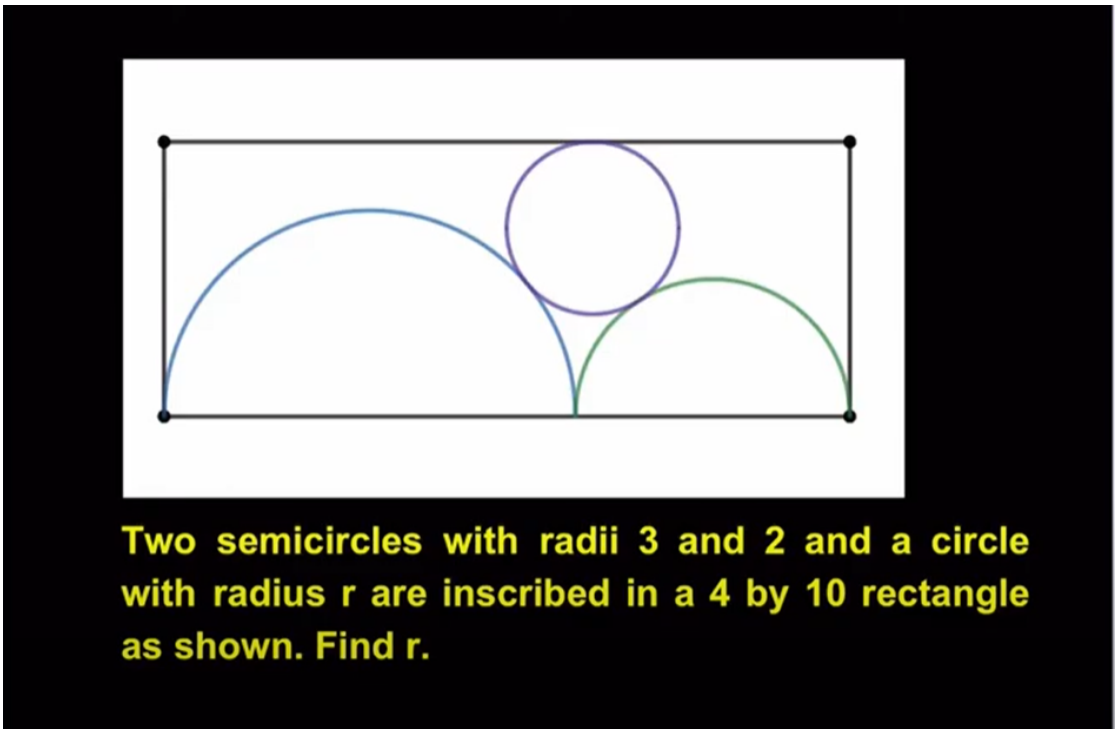
Problem 1.17. Find the value of x



Problem 1.18. Find the radius of the shaded circle



Problem 1.19. Find the radius of the circle



Problem 1.20. Find the radius of the shaded circle